

## TO HONOR STEVENS AND REPEAL HIS LAW (FOR THE AUDITORY SYSTEM)

Mary Florentine<sup>1,2</sup> and Michael Epstein<sup>1,2,3</sup>

<sup>1</sup>*Institute for Hearing, Speech, and Language*

<sup>2</sup>*Dept. Speech-Language Pathology and Audiology (133 FR)*

<sup>3</sup>*Communications and Digital Signal Processing Center,  
Dept. of Electrical and Computer Engineering (442 DA)*

*Northeastern University, Boston, MA 02115 USA*

### Abstract

*The purpose of this paper is twofold: to honor S.S. Stevens and his efforts to further our understanding of the psychophysical law for loudness, and to propose a modification of this law in the light of converging evidence. In order to provide a simple, comprehensive law to fit the data from different modalities, second-order details in the loudness function were ignored. These “details” are essential for a full understanding of how loudness grows with increasing intensity. Data from a number of experiments reviewed in this paper indicate that a power function is only a rough approximation to the loudness growth function, because a power law does not fit the data at low and moderate levels. This paper proposes a new psychophysical law for loudness to better describe the data. A non-stationary point of inflection law [or an inflected exponential (INEX) law] appears to be the best description of currently available data.*

### Background

On the 100<sup>th</sup> anniversary of Fechner’s (1860) book, the *Elemente der Psychophysik*, S. S. Stevens presented a paper entitled, “To Honor Fechner and Repeal His Law” at a joint meeting of the American Psychological Meeting and the Psychometric Society in Chicago, USA. This paper was published in *Science* the following year (Stevens, 1961) and sought to inform “...a power function, and not a log function, describes the operating characteristic of a sensory system.” As we honor Stevens today for his contributions to psychophysics, we also wish to build on his observations—as he built on Fechner’s—and present a new psychophysical law for loudness. We wish to dedicate this paper in honor and memory of Søren Buus, who worked with the authors on the early development of this new law and who would often say “I believe this is correct today,” because he always wanted to leave room for changing his mind tomorrow in the light of new data.

Fechner and Stevens had much in common. As it was Fechner’s version of the psychophysical law that made him famous, it was Stevens’ Power Law that became his primary claim to fame. With it he founded modern psychophysics and he became famous for reasons similar to Fechner. He had a mathematical formula (a law) to test the relationship between the physical stimulus and perception. He honed his psychophysical tools (*i.e.*, magnitude estimation and production) to test his theory. He had a laboratory that educated many students who became the disciples of his thoughts, and he published and promoted his ideas with great skill and vigor.

Stevens' ability to express his ideas in words, his energy to push an idea forward, and his position as the director of the Psychological Laboratories at Harvard University in Cambridge, Massachusetts ensured that his ideas spread quickly. In fact, all the newly emerging texts in psychophysics described the comprehensive "Power Law." But, even as Stevens (1962) wrote of the "Surprising Simplicity of Sensory Metrics," serious questions about the application of this new theory to loudness were evident. At the very least, there were clear indications that a simple power function was only a first approximation to the data. For example, there were indications that a power law needed to be modified to fit the data near threshold (Hellman and Zwischlocki, 1961). Later, other changes would become obvious.

The purpose of this paper is to describe two deviations to the power law as they relate to the auditory system. In fact, these "deviations" result in an entirely different psychophysical law. This new law should not be surprising because some of the data that point to it have been present for decades. We now see them with new eyes. The first deviation is that the loudness function becomes steep as it approaches threshold. The second deviation is that the loudness function is less steep at moderate levels than at low and high levels. Each of these deviations will be discussed in turn.

### **Deviations from a Power Law**

The loudness function at and near threshold is steeper than at moderate levels. Early questioning of the power law for audition came from Zwischlocki and Hellman's (1960) talk presented at to the Acoustical Society of America in Providence, RI and was published the following year (Hellman and Zwischlocki, 1961). They pointed out that loudness at threshold did not follow a simple power law and the slope of the loudness function changed below 30 dB SL. According to a summary figure from Buus, Müsch, and Florentine (1998), this contention is supported by a number of studies that measured the loudness of low-level tones using a variety of methods (*i.e.*, Robinson, 1957; Zwicker, 1958; Feldtkeller *et al.*, 1959; Hellman and Zwischlocki, 1961; Hellman and Zwischlocki, 1963; Scharf, 1961; Canévet *et al.*, 1986; Marks, 1978; Marks, 1979; Buus *et al.*, 1998). Near threshold, the average slope is about unity or slightly larger. As level increases, the slope decreases to 0.36 at 20 dB SL, and 0.19 at 40 dB SL. Although the slope of 0.19 at 40 dB SL is shallower than the slope of about 0.3 usually used to describe the loudness function at moderate and high levels for 1-kHz tones, it is in excellent agreement with the shallow mid-level slopes of the loudness functions derived from temporal-integration data that will be discussed later (see Buus *et al.*, 1997).

The loudness function is less steep at moderate levels than at low and high levels. Data supporting this concept come from spectral integration of loudness, temporal integration of loudness, and binaural loudness summation. The number of different methods of deriving the loudness function provides converging evidence of a nonmonotonic loudness function. Not all of the methods were suitable for testing all levels. For example, spectral integration of loudness was measured to assess the loudness growth function at levels of 43 dB SPL and lower. The method used to do this was based Fletcher and Munson's (1933) assumption that the total loudness of a complex sound will be equal to the sum of the loudnesses of the individual component tones, provided mutual masking does not occur. For levels lower than about 45 dB SPL and for widely spaced components, mutual masking does not cause a problem. Accordingly, a low-level tone complex consisting of  $n$  widely separated, equally loud tones should have a total loudness equal to  $n$  times the loudness of a single component tone. The derived loudness function obtained using this method are shown in Fig. 1 by the thick, continuous line [Inflected Exponential Function (Part I)] and are in excellent agreement with the loudness data at and near threshold described previously (see Buus *et al.*, 1998).

To assess the loudness function from about 23 dB SL to 100 dB SPL, Florentine *et al.*

(1996) derived the loudness function using measurements of temporal integration of loudness and their results are shown in Fig. 1 by the thin, continuous line [Inflected Exponential Function (Part II)]. They measured the effect of level on the amount of temporal integration for loudness, defined as the level difference between equally loud short and long tones. [Although temporal integration of loudness had been measured, Florentine *et al.* (1996) used a modern psychophysical procedure, included control conditions to account for bias, and tested a large range of levels that permitted assessment of the shape of the function.] Their results agreed with the majority of available data and showed that the amount of temporal integration for loudness varied nonmonotonically with level and was greatest at moderate levels. On the basis of these data, Florentine *et al.* suggested that loudness might grow more slowly at moderate than at low and high levels. The logic behind this observation was straightforward. If the vertical distance between loudness functions for short and long tones (plotted on a log scale as a function of level) is independent of level, then the loudness growth functions for these stimuli must be shallower at moderate levels than at low and high levels. However, at the time, the existence of a constant vertical distance between the functions (*i.e.*, the Equal-Loudness-Ratio Hypothesis) needed to be tested. Since that time, several experiments have supported this hypothesis for levels exceeding 40 dB SPL (see Epstein and Florentine, 2005). Therefore, loudness functions for short and long tones—around 1-kHz, at least—are parallel when plotted on a log scale and are shallower at moderate levels than at low and high levels.

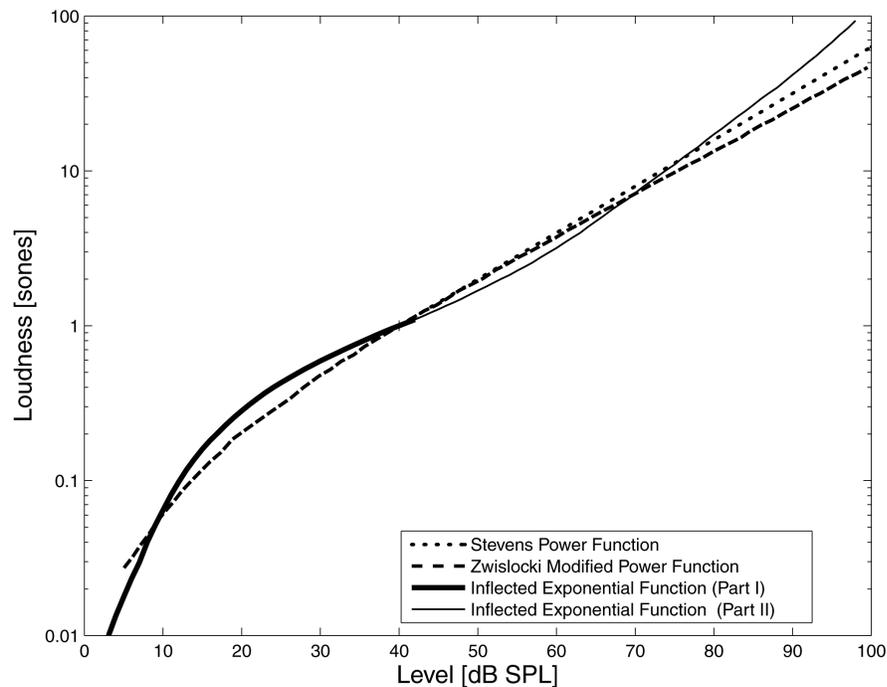
The difference between the inflected exponential function, shown in Fig. 1, and Stevens' power function [or Zwislocki's (1965) modified power function] is small at moderate levels, but clearly present. The difference between the functions can be made to look even smaller by expanding the ordinate, as was common practice by S. S. Stevens and others.

## Discussion and Conclusion

It is well accepted that the loudness function becomes steep near threshold. Although a simple power function provides a reasonable approximation to the loudness of mid-frequency tones at moderate and high levels, to be consistent with temporal integration of loudness data, there must be an allowance for a change in the slope of the function at moderate levels. In addition, careful inspection of loudness functions that were obtained with a wide enough range of levels and sufficient data show a flattening in the function at moderate levels. For example, a mid-level flattening is even apparent in the loudness functions obtained by Fletcher and Munson (1933).

Whereas a power law is a better description of the loudness function than a logarithmic law, a non-stationary point of inflection law [or an inflected exponential (INEX) law] appears to be the best description of currently available data. The INEX law also suitably covers the three major segments of the function. At low levels (less than about 25-30 dB SPL), loudness grows more rapidly. At moderate levels, loudness shows compressive growth (about 25-60 dB SPL). At high levels (greater than about 60 dB SPL), the rate of loudness growth again increases. It seems clear that a power law for audition should be repealed in favor of an INEX law, at least for the moment.

Despite that fact that Stevens promoted the power law for loudness, in at least one paper published close to the end of his life (*i.e.*, Stevens, 1972), he argued that the loudness function for a 1-kHz tone deviated from a simple power function and alluded to "systematic deviations from a strict power function". Perhaps he did not have enough time to modify the psychophysical law by the time he discovered the deviation. We like to think that he was pointing in that direction and would be pleased with the result of this presentation. After all,



**Figure 1. Stevens' Power Function is plotted with each of the modifications presented in this paper to arrive at the Inflected Exponential (INEX) function. The INEX function was taken from Buus and Florentine (2001).**

science is a changing story. In the light of new insights, a better law may become evident. We think that S. S. Stevens and Søren Buus would agree!

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